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BOOK REVIEW

Potential Flows of Viscous and Viscoelastic Fluids. By DANIEL JOSEPH, TOSHIO FUNADA & JING WANG. Cambridge University Press, New York, 2007. 516 pp. ISBN-13: 978 0 521 87337 6 $104.99.

Perhaps the central intellectual development underlying the modern theory of fluid motion is the idea that the solution for a complex flow field can be broken down into pieces, each of which has a different set of dominant forces or 'dominant balances'. The discovery of this idea is due to Prandtl, who realized that inviscid solutions to the Navier–Stokes equation could satisfy the no-slip boundary condition as long as a very thin boundary layer were present near the solid. Outside of the boundary layer, viscous forces are vanishingly small – typical solutions are given by inviscid potential flow – whereas inside the boundary layer, viscous forces are large and bring the fluid flow to rest at the solid. One of the reasons which makes Prandtl's idea so important is that it applies to a broad array of problems in well beyond fluid mechanics per se. But even within fluid mechanics, dominant balances arise in many different guises. Indeed, entire branches of the subject have been organized around their relevant dominant balances – be they high or low Reynolds number, super or subsonic, capillary-dominated, interfacial and the like. In *Potential Flows of Viscous and Viscoelastic Fluids*, Joseph, Funada and Wang make their case that viscous potential flows should provide another important paradigm for organizing a variety of fluid motions.

Where do viscous potential flows arise? If $\boldsymbol{v} = \nabla \phi$, and $\nabla \cdot \boldsymbol{v} = 0$, then $\nabla^2 \boldsymbol{v} = 0$. Therefore any potential flow solution also satisfies the viscous Navier–Stokes equations. Using this observation as their paradigm, the authors summarize many problems in classical fluid mechanics and beyond. The problems in the first part of the book include: (i) rising bubbles; (ii) the Rayleigh–Taylor instability of viscous fluids; (iii) the Kelvin–Helmholtz instability; (iv) damping of waves by viscosity; (v) irrotational Faraday waves; (vi) liquid jet stability; (vii) cavitation. The second part of the book contains: (i) liquid jets in high-Mach-number streams; (ii) irrotational flows of viscoelastic fluids; (iii) irrotational stability theory of viscoelastic fluids.

The first part of the book contains, almost exclusively, potential flows bounded by free surfaces, whereas the second part of the book pushes contains topics going outside of the standard incompressible Navier–Stokes equation. In each of the chapters, the authors tersely summarize the basic theory of each phenomenon and then provide a reinterpretation, often supplemented by figures and calculations from their own research papers. Much of this is quite tough reading owing to the technical nature of the calculations as well as the brief figure captions. To illustrate the latter, I opened the book to three random pages: from the caption to figure 8.1, I had trouble understanding the difference between the different plots; while the caption to figure 16.2 was quite clear, figure 18.16 contains six subfigures which are quite similar to each other and whose point was therefore obscured.

The book contains some interesting nuggets that I had not been previously aware of. For example, it is argued that the Rayleigh–Plesset equation for the oscillations of a viscous bubble would be better called the Rayleigh–Poritsky equation, since Plesset's 1949 paper does not actually include the effects of viscosity, whereas Poritsky does.

(Indeed, further investigation led me to Lauterborn's (Lauterborn 1976) suggestion that the equation should be called the RPNNP equation, after the contributors Rayleigh, Plesset, Noltingka, Neppiras and Poritsky.) The authors have developed a nice method for computing viscous boundary layers near a free surface (nicely outlined in chapter 12), which they appear to demonstrate is quite accurate. They compute a viscous pressure in the boundary layer, and use this to act on the normal stress equation, hence coupling viscous dissipation to oscillations of the surface. They call their method 'viscous correction to viscous potential flow' (VCVPF), and it gives excellent approximations to a host of problems, including Lamb's solution to the decay rate due to viscosity for long waves on a free surface. It does much better than just viscous potential flow (VPF) alone; the latter refers to the practice of evaluating viscous normal stresses on an interface from an irrotational theory alone. We learn that VCVPF reproduces Lamb's dissipation method, though only the former explicitly calculates pressure corrections in the boundary layer. These different ideas are critically applied to many of the rich list of problems discussed in the book.

Should viscous potential flows be entered into the lexicon as one of the organizing pillars of fluid mechanics? I remain unconvinced. Many of the flows satisfying this description are also elegantly and productively classified in other ways. For example, consider the subject of the largest part of the book, flows with free surfaces, like ocean waves or bubble oscillations. (Viscous) potential flow solutions indeed typically apply far away from a free surface. At the free surface, instead of the no-slip boundary condition, the boundary has no tangential stress. For most surface shapes, like waves on the ocean, a vanishing tangential stress implies a non-zero vorticity at the surface. This then sets up a thin boundary layer near the free surface where viscous stresses are important. The mathematical structure of these flows are thus commonly and profitably compared with other boundary-layer flows, like those of Prandtl. The advantage of this classification is that it emphasizes the mathematical and structural similarity of the flow.

But, classifications are more a matter of taste than science, and to that end others might be more convinced by the viscous potential flow paradigm than I was. This book will be of interest to anyone interested in an advanced introduction to the topics in the book, as well as to those interested in reconsidering how we classify flows. For any reader, the book provides yet another reminder of the tremendous richness of the solutions to the Navier–Stokes equation: even simple (or perhaps even exact) dominant balances like that of the potential flow solution arise ubiquitously, and that by using them effectively we can obtain detailed, approximate and highly accurate understanding of the flows.

REFERENCE

Lauterborn, W. 1976 Numerical investigation of nonlinear oscillations of gas bubbles in liquids. *J. Acoust. Soc. Am.* **59**, 283–293.

Michael P. Brenner
School of Engineering and Applied Sciences, Harvard University